From:	jwjones4@pacbell.net
To:	Bennett, Jim
Subject:	Erf large u approx
Date:	Wednesday, July 31, 2013 4:40:25 PM
Attachments:	Well Fcn Approx in Excel.xlsx Theis Large u Spreadsheet 2-8-08.xlsx well fcn large u approx.pdf

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Jim,

Here's the approximation for u>1. see attached. its instructive to compare the behavior of the three calculations near u=0.1

>copy of formula from Math Function handbook (Abramowithz and Stegun, 1970)

>table showing erf calc, eqn 5.1.56 (eqn 5.1.53 is the one we see used in the Jacob approx.)

>old spreadsheet where I ran both calculations, checked u value, then picked the function to use. crude, I'd suggest doing the same but with an IF function based on u

should plug in fairly easily...have fun!

Jay

Jay W. Jones PG, PGP, Ph.D
Environmental Navigation Services, Inc.
760 944 9576

Drawdown, at a point (Theis semi-log approximation)

T	S	DD, ft	% 500ft
67	0.0100		0.0%
67	0.0001		0.0%
167	0.0100	1.75	0.4%
267	0.0100		0.0%
267	0.0001		0.0%

'(+DD, -WL inc)

cells with input values, all use defined va

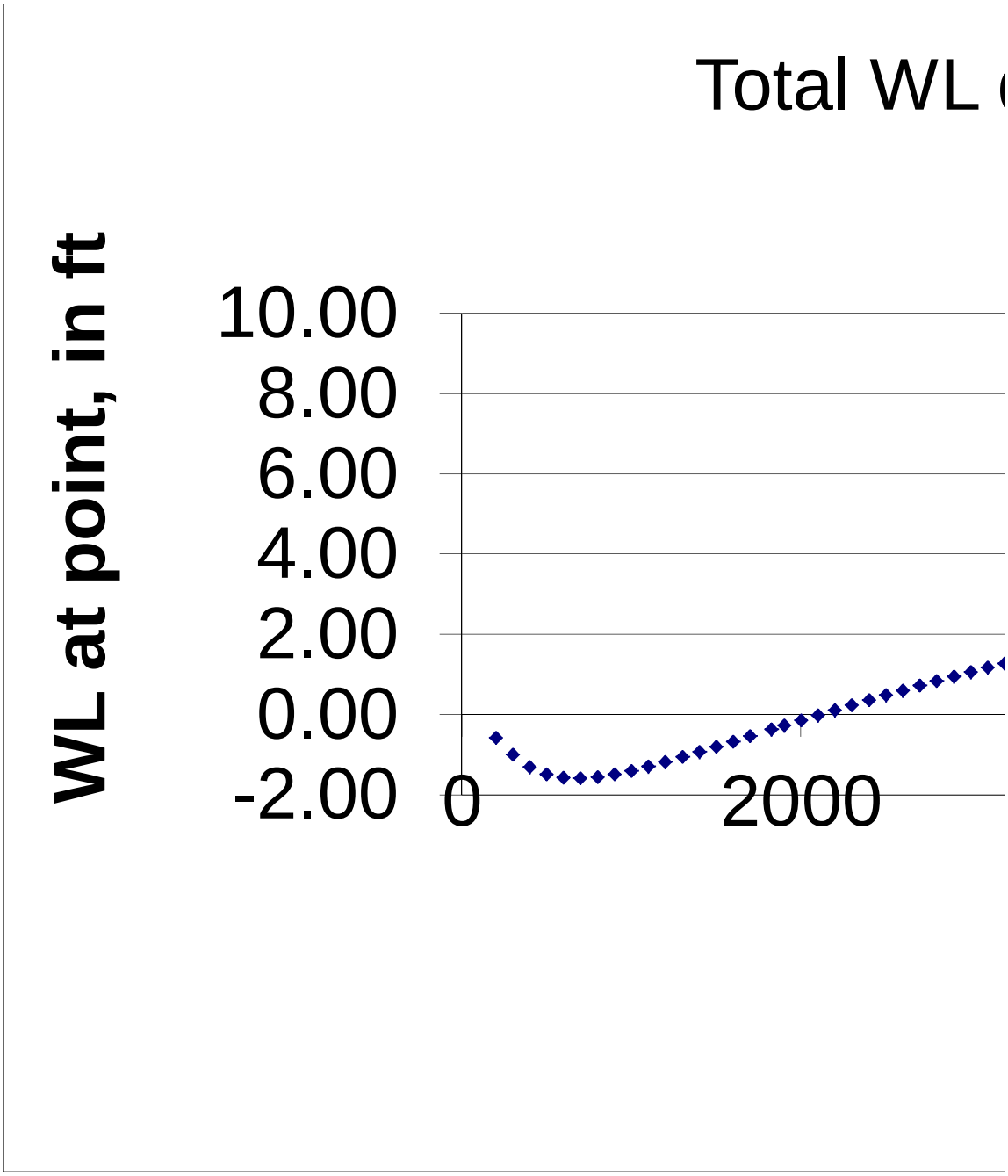
167 Transmissivity, ft2/day

1.00E-02 Storativity, dimensionless

	P1 Wells on Pauma Ck		P2 New Prod Well (N)	
ft from pt.	dist1	7100	dist2	4900
acft/yr	Q1	175	Q2	59
ft3/day		20885		7041

time, days	Total WL change		WL change	u	WL change	u
1825	-0.36	(5 years)	6.77	0.41350	4.14	0.19695
3650	1.75	(10 years)	11.90	0.20675	6.16	0.09847
50	-95300.23		-95292.43	15.09	-12.94	7.19
75	-1351.76		-1351.63	10.06	-0.17	4.79
100	-64.41		-64.32	7.55	0.01	3.59
200	-0.57		0.01	3.77320	0.22	1.79716
300	-0.99		0.24	2.51547	0.53	1.19810
400	-1.30		0.57	1.88660	0.88	0.89858
500	-1.48		0.98	1.50928	1.21	0.71886
600	-1.57		1.44	1.25773	1.53	0.59905
700	-1.58		1.92	1.07806	1.82	0.51347
800	-1.55		2.40	0.94330	2.10	0.44929
900	-1.48		2.89	0.83849	2.36	0.39937
1000	-1.39		3.36	0.75464	2.60	0.35943
1100	-1.29		3.82	0.68604	2.83	0.32676
1200	-1.17		4.27	0.62887	3.04	0.29953
1300	-1.05		4.70	0.58049	3.24	0.27649
1400	-0.92		5.13	0.53903	3.43	0.25674
1500	-0.79		5.53	0.50309	3.62	0.23962
1600	-0.66		5.93	0.47165	3.79	0.22464
1700	-0.53		6.31	0.44391	3.95	0.21143
1825	-0.36		6.77	0.41350	4.14	0.19695
1900	-0.27		7.04	0.39718	4.26	0.18917
2000	-0.14		7.38	0.37732	4.40	0.17972
2100	-0.01		7.72	0.35935	4.54	0.17116
2200	0.12		8.05	0.34302	4.67	0.16338
2300	0.25		8.36	0.32810	4.80	0.15627
2400	0.37		8.67	0.31443	4.92	0.14976
2500	0.49		8.97	0.30186	5.04	0.14377
2600	0.61		9.26	0.29025	5.15	0.13824
2700	0.73		9.54	0.27950	5.26	0.13312
2800	0.84		9.82	0.26951	5.37	0.12837
2900	0.96		10.08	0.26022	5.47	0.12394
3000	1.07		10.34	0.25155	5.57	0.11981
3100	1.18		10.60	0.24343	5.67	0.11595

3200	1.28	10.85	0.23583	5.77	0.11232
3300	1.39	11.09	0.22868	5.86	0.10892
3400	1.49	11.33	0.22195	5.95	0.10572
3500	1.60	11.56	0.21561	6.04	0.10269
3650	1.75	11.90	0.20675	6.16	0.09847
5000	2.94	14.52	0.15093	7.13	0.07189
6000	3.68	16.10	0.12577	7.71	0.05991
8000	4.91	18.67	0.09433	8.62	0.04493
9000	5.43	19.74	0.08385	9.00	0.03994
10000	5.90	20.71	0.07546	9.34	0.03594



existing wells pump 60, proposed pump 294, difference of 191
therefore P1 should = 235 and P2 = 59

ariable names

Casino Recharge from irrigation and wwtp = 37, proposed = 178, increase o
Pauma Creek recharge adjusted to 0 if take all diversions or divert to = 5% i
Grove Irrigation Recharge - not used

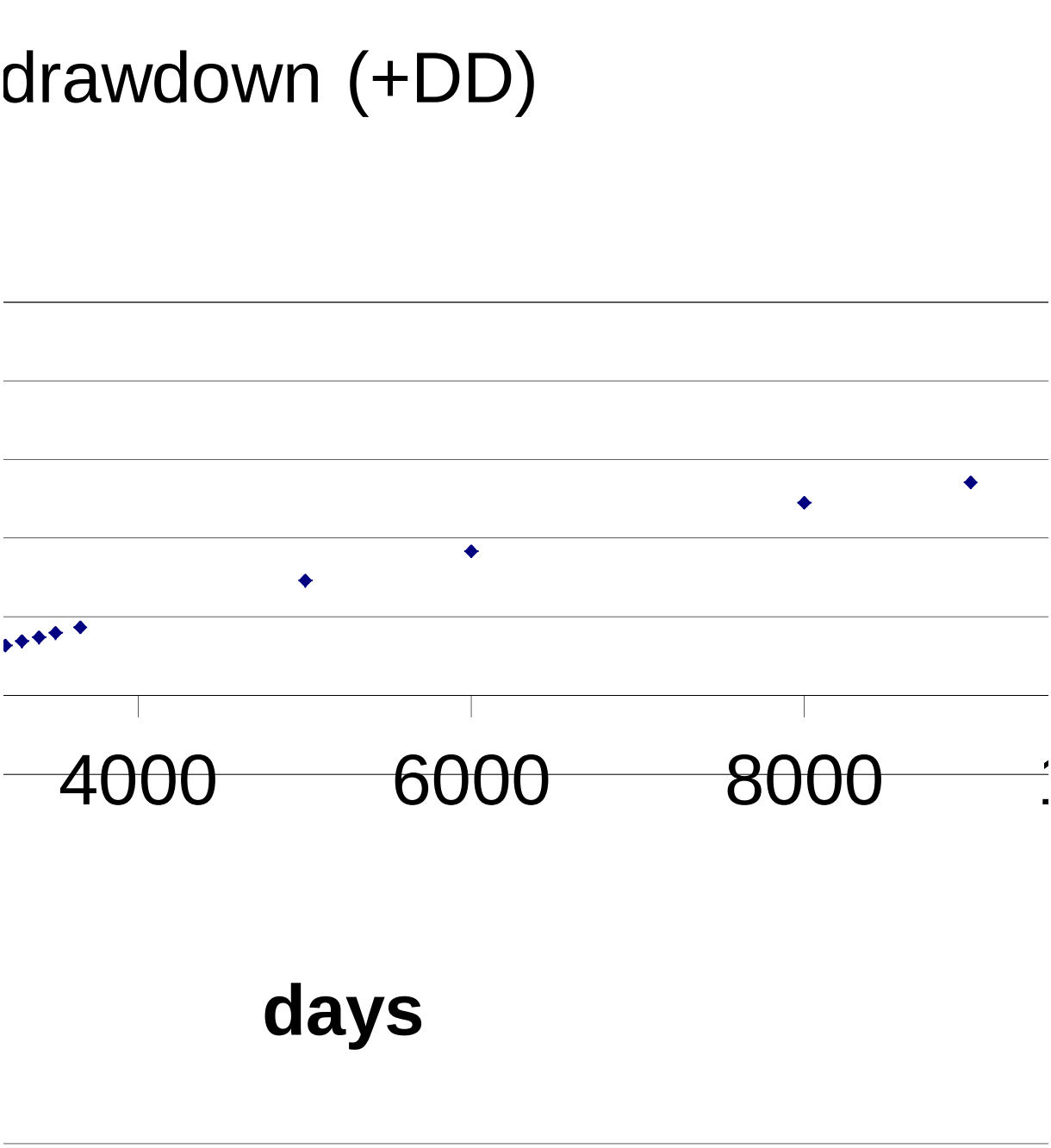
R1	
Casino Recharge	
dist3	4500
Q3	-144
	-17185

R2	
Pauma Ck Recharge	
dist4	3134
Q4	0
	0

approx for 1<u<inf.
Handbook of Mathematical Functions
Abramowitz and Stegun, 1970
p. 231 no.5.1.56

WL change	u	WL change	u	WL change-u>>1	W(u)	numer.	denom.
-11.28	0.17	0.00	0.08				
-16.32	0.08	0.00	0.04	0.00	3.172889	0.645478	4.850685
5.14	6.06	0.00	2.94	0.00	0.014072	474.637	605.9087
0.04	4.04	0.00	1.96	0.00	0.051657	165.9764	230.748
-0.10	3.03	0.00	1.47	0.00	0.104541	83.93328	125.5043
-0.80	1.52	0.00	0.74	0.00	0.349844	20.07519	37.42059
-1.77	1.01	0.00	0.49	0.00	0.572005	9.904939	21.64196
-2.75	0.76	0.00	0.37	0.00	0.760347	6.326043	15.67174
-3.67	0.61	0.00	0.29	0.00	0.921748	4.594198	12.63085
-4.53	0.51	0.00	0.25	0.00	1.062742	3.598084	10.81298
-5.32	0.43	0.00	0.21	0.00	1.188177	2.959691	9.612091
-6.05	0.38	0.00	0.18	0.00	1.301547	2.519196	8.762937
-6.73	0.34	0.00	0.16	0.00	1.405395	2.198554	8.1322
-7.35	0.30	0.00	0.15	0.00	1.501603	1.955529	7.645944
-7.94	0.28	0.00	0.13	0.00	1.59159	1.765424	7.260015
-8.49	0.25	0.00	0.12	0.00	1.676447	1.612907	6.946488
-9.00	0.23	0.00	0.11	0.00	1.757022	1.487985	6.686867
-9.48	0.22	0.00	0.11	0.00	1.833987	1.383887	6.468432
-9.94	0.20	0.00	0.10	0.00	1.90788	1.295869	6.282155
-10.38	0.19	0.00	0.09	0.00	1.979137	1.220513	6.121456
-10.79	0.18	0.00	0.09	0.00	2.048116	1.1553	5.981428
-11.28	0.17	0.00	0.08	0.00	2.131581	1.085197	5.829864
-11.56	0.16	0.00	0.08	0.00	2.180371	1.048148	5.749308
-11.92	0.15	0.00	0.07	0.00	2.244101	1.003619	5.65206
-12.26	0.14	0.00	0.07	0.00	2.306476	0.963847	5.56479
-12.59	0.14	0.00	0.07	0.00	2.367648	0.928113	5.486043
-12.91	0.13	0.00	0.06	0.00	2.427743	0.895837	5.414631
-13.22	0.13	0.00	0.06	0.00	2.486873	0.866543	5.349578
-13.51	0.12	0.00	0.06	0.00	2.545134	0.83984	5.290073
-13.80	0.12	0.00	0.06	0.00	2.602609	0.815399	5.235436
-14.08	0.11	0.00	0.05	0.00	2.659372	0.792947	5.185096
-14.34	0.11	0.00	0.05	0.00	2.715486	0.772252	5.138565
-14.60	0.10	0.00	0.05	0.00	2.77101	0.753116	5.095428
-14.85	0.10	0.00	0.05	0.00	2.825993	0.73537	5.055327
-15.09	0.10	0.00	0.05	0.00	2.88048	0.71887	5.017954

-15.33	0.09	0.00	0.05	0.00	2.934511	0.703488	4.983038
-15.56	0.09	0.00	0.04	0.00	2.988122	0.689116	4.950347
-15.78	0.09	0.00	0.04	0.00	3.041345	0.675657	4.919674
-16.00	0.09	0.00	0.04	0.00	3.094209	0.663027	4.890839
-16.32	0.08	0.00	0.04	0.00	3.172889	0.645478	4.850685
-18.72	0.06	0.00	0.03	0.00	3.857365	0.537529	4.601379
-20.13	0.05	0.00	0.02	0.00	4.347465	0.490344	4.49109
-22.38	0.04	0.00	0.02	0.00	5.306518	0.432626	4.355007
-23.31	0.03	0.00	0.02	0.00	5.779911	0.413696	4.310084
-24.15	0.03	0.00	0.01	0.00	6.250771	0.398663	4.274301



f 141 = R1
npact

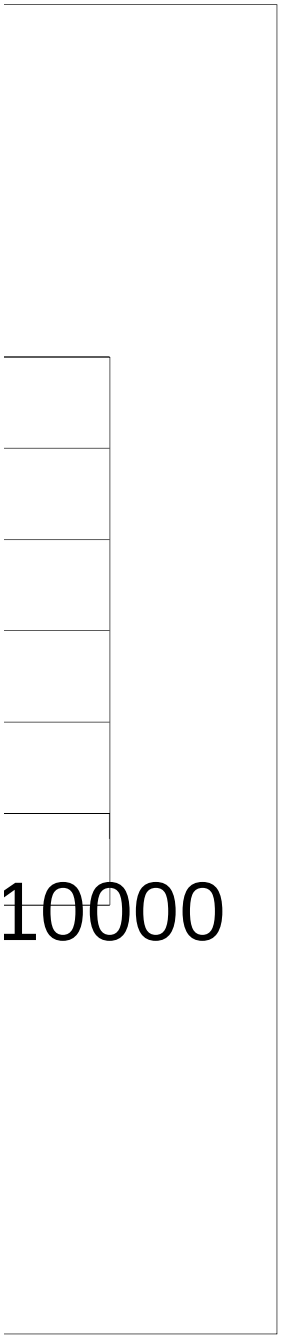
Does R1 combine increase in wwtp and landscape minue c
If so R1 adjsuted to show irrigation returns from new use of

R3
Grove Irrigation Recharge (dec)

dist5	1912
Q5	0
	0

WL change	u
0.00	0.030
0.00	0.015
0.00	1.095
0.00	0.730
0.00	0.547
0.00	0.274
0.00	0.182
0.00	0.137
0.00	0.109
0.00	0.091
0.00	0.078
0.00	0.068
0.00	0.061
0.00	0.055
0.00	0.050
0.00	0.046
0.00	0.042
0.00	0.039
0.00	0.036
0.00	0.034
0.00	0.032
0.00	0.030
0.00	0.029
0.00	0.027
0.00	0.026
0.00	0.025
0.00	0.024
0.00	0.023
0.00	0.022
0.00	0.021
0.00	0.020
0.00	0.020
0.00	0.019
0.00	0.018
0.00	0.018

0.00	0.017
0.00	0.017
0.00	0.016
0.00	0.016
0.00	0.015
0.00	0.011
0.00	0.009
0.00	0.007
0.00	0.006
0.00	0.005



lecrease in groves? = 52-20 = 32
212 diversions = 139 more return flows

Drawdown, at a point (Theis semi-log approximation)

T	S	DD, ft	% 500ft
67	0.0100	0.04	0%
67	0.0001	77.33	15%
167	0.0100	3.01	1%
267	0.0100	3.49	1%
267	0.0001	26.71	5%

'(+DD, -WL inc)

cells with input values, all use defined va

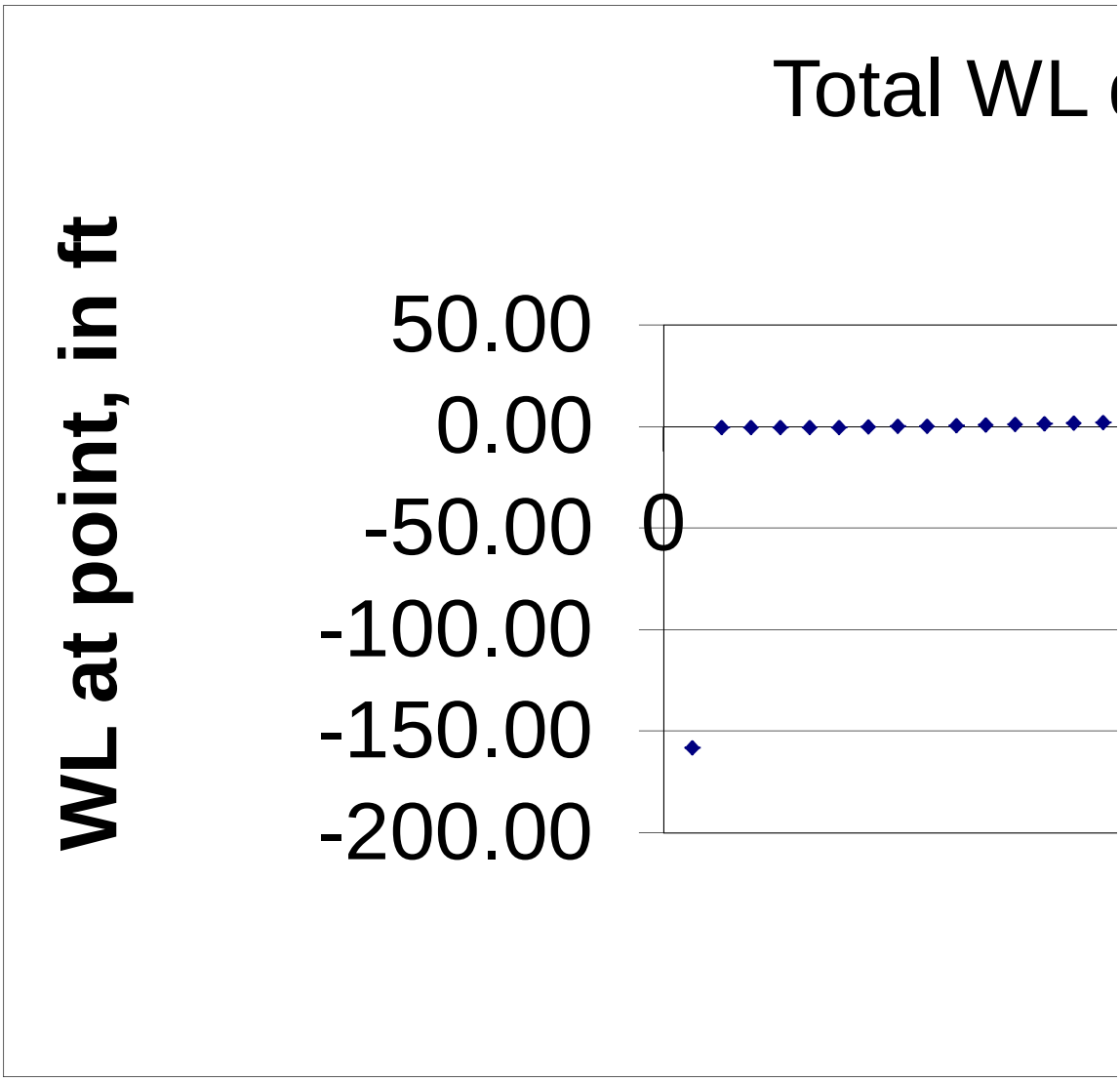
167 Transmissivity, ft2/day

1.00E-02 Storativity, dimensionless

	P1 Wells on Pauma Ck		P2 New Prod Well (N)	
ft from pt.	dist1	7300	dist2	6400
acft/yr	Q1	235	Q2	59
ft3/day		28045		7041

time, days	Total WL change		WL change	u	WL change	u
1825	3.01	(5 years)	8.61	0.43713	2.76	0.33599
50	-231804.93		-228505.36	15.96	-3655.42	12.26
75	-3306.09		-3260.15	10.64	-50.73	8.18
100	-157.96		-155.78	7.98	-2.38	6.13
200	-0.28		-0.04	3.98877	0.04	3.06587
300	-0.36		0.27	2.65918	0.15	2.04391
400	-0.43		0.66	1.99439	0.32	1.53293
500	-0.38		1.16	1.59551	0.51	1.22635
600	-0.25		1.73	1.32959	0.71	1.02196
700	-0.05		2.33	1.13965	0.91	0.87596
800	0.18		2.95	0.99719	1.11	0.76647
900	0.43		3.56	0.88639	1.30	0.68130
1000	0.71		4.17	0.79775	1.48	0.61317
1100	0.99		4.76	0.72523	1.66	0.55743
1200	1.27		5.34	0.66480	1.83	0.51098
1300	1.56		5.91	0.61366	2.00	0.47167
1400	1.84		6.46	0.56982	2.16	0.43798
1500	2.12		6.99	0.53184	2.31	0.40878
1600	2.40		7.50	0.49860	2.45	0.38323
1700	2.67		8.00	0.46927	2.59	0.36069
1825	3.01		8.61	0.43713	2.76	0.33599
1900	3.20		8.96	0.41987	2.86	0.32272
2000	3.46		9.41	0.39888	2.98	0.30659
2100	3.71		9.85	0.37988	3.11	0.29199
2200	3.96		10.28	0.36262	3.22	0.27872
2300	4.20		10.70	0.34685	3.34	0.26660
2400	4.44		11.10	0.33240	3.45	0.25549
2500	4.67		11.50	0.31910	3.55	0.24527
2600	4.89		11.88	0.30683	3.66	0.23584
2700	5.11		12.25	0.29546	3.76	0.22710
2800	5.33		12.62	0.28491	3.86	0.21899
2900	5.54		12.97	0.27509	3.95	0.21144
3000	5.75		13.32	0.26592	4.04	0.20439
3100	5.95		13.66	0.25734	4.13	0.19780
3200	6.15		13.99	0.24930	4.22	0.19162

3300	6.34	14.31	0.24174	4.31	0.18581
3400	6.53	14.62	0.23463	4.39	0.18035
3500	6.72	14.93	0.22793	4.47	0.17519
3600	6.90	15.23	0.22160	4.55	0.17033
3700	7.08	15.52	0.21561	4.63	0.16572
3800	7.25	15.81	0.20994	4.70	0.16136
3900	7.43	16.09	0.20455	4.78	0.15722
4000	7.60	16.37	0.19944	4.85	0.15329
5000	9.14	18.86	0.15955	5.50	0.12263



existing wells pump 60, proposed pump 294, difference of 191
therfore P1 should = 235 and P2 = 59

ariable names

Casino Recharge from irrigation and wwtp = 37, proposed = 178, increase o
Pauma Creek recharge adjusted to 0 if take all diversions or divert to = 5% i
Grove Irrigation Recharge - not used

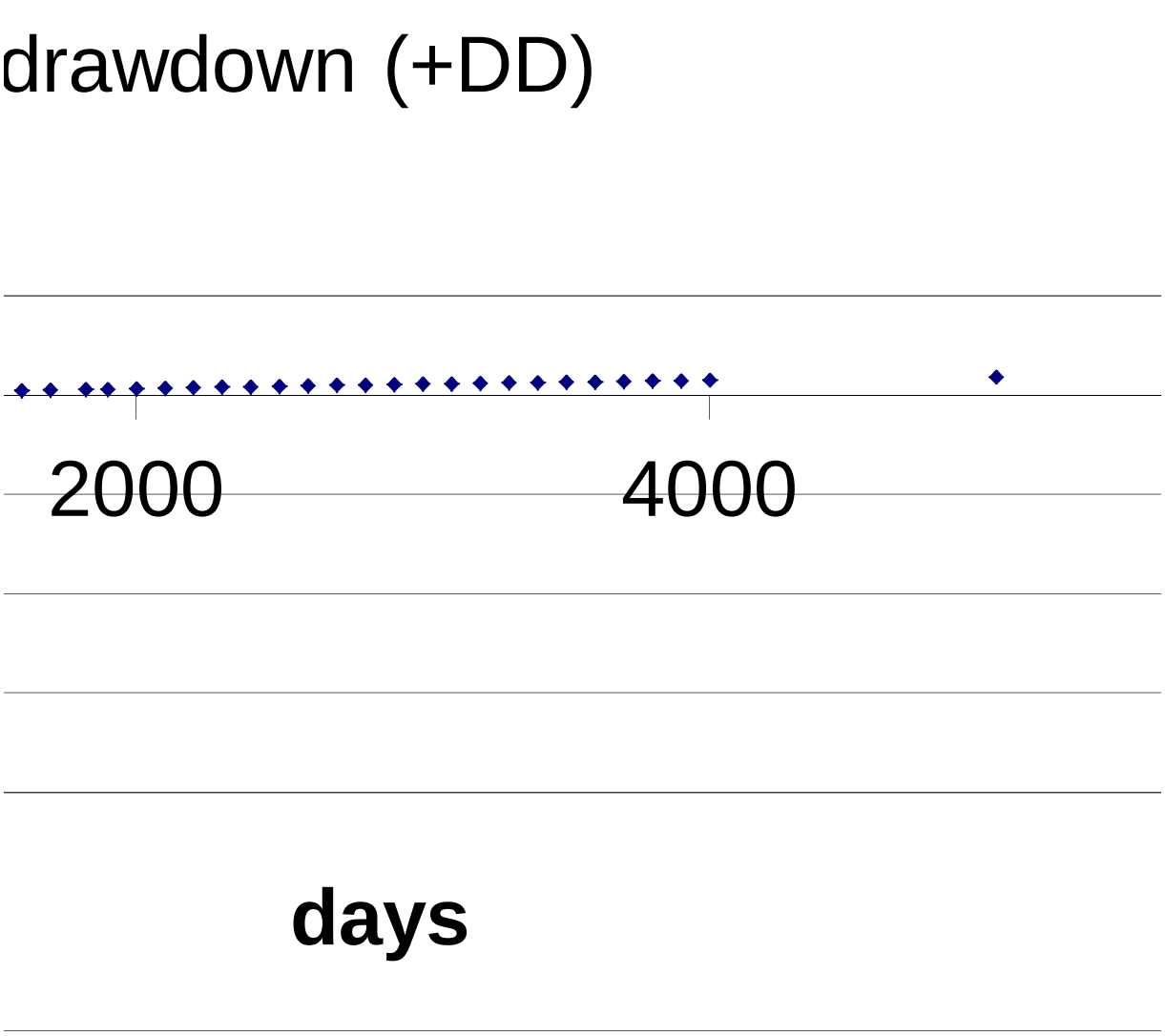
R1	
Casino Recharge	
dist3	5500
Q3	-140
	-16708

R2	
Pauma Ck Recharge	
dist4	11600
Q4	0
	0

approx for 1<u<inf.
Handbook of Mathematical Functions
Abramowitz and Stegun, 1970
p. 231 no.5.1.56

WL change	u	WL change	u	WL change-u>>1	W(u)	numer.	denom.
-8.36	0.25	0.00	1.10				
355.85	9.06	0.00	40.29	0.00	7.72E-20	3224643	3302831
4.79	6.04	0.00	26.86	0.00	7.78E-14	699735.7	724912.2
0.20	4.53	0.00	20.14	0.00	8.46E-11	242226.3	253728
-0.27	2.26	0.00	10.07	0.00	3.84E-06	20969.19	22888.42
-0.79	1.51	0.00	6.71	0.00	0.000159	5500.541	6232.119
-1.41	1.13	0.00	5.04	0.00	0.001101	2239.83	2626.079
-2.05	0.91	0.00	4.03	0.00	0.00365	1152.21	1394.434
-2.69	0.75	0.00	3.36	0.00	0.008322	684.2751	853.0205
-3.30	0.65	0.00	2.88	0.00	0.015254	447.5399	573.6479
-3.88	0.57	0.00	2.52	0.00	0.024333	313.5706	412.6296
-4.43	0.50	0.00	2.24	0.00	0.03532	231.2814	312.0232
-4.95	0.45	0.00	2.01	0.00	0.047939	177.4799	245.1847
-5.44	0.41	0.00	1.83	0.00	0.061912	140.535	198.591
-5.91	0.38	0.00	1.68	0.00	0.07699	114.1432	164.8295
-6.35	0.35	0.00	1.55	0.00	0.092949	94.66807	139.5781
-6.77	0.32	0.00	1.44	0.00	0.109601	79.9022	120.1859
-7.17	0.30	0.00	1.34	0.00	0.126787	68.44696	104.9572
-7.56	0.28	0.00	1.26	0.00	0.144372	59.38362	92.76744
-7.92	0.27	0.00	1.18	0.00	0.162246	52.08965	82.84774
-8.36	0.25	0.00	1.10	0.00	0.184857	44.81267	72.83336
-8.61	0.24	0.00	1.06	0.00	0.198509	41.20091	67.8114
-8.94	0.23	0.00	1.01	0.00	0.21676	37.07257	62.02233
-9.25	0.22	0.00	0.96	0.00	0.235019	33.58011	57.07864
-9.55	0.21	0.00	0.92	0.00	0.253242	30.59799	52.81897
-9.84	0.20	0.00	0.88	0.00	0.271396	28.03022	49.11901
-10.12	0.19	0.00	0.84	0.00	0.289454	25.80236	45.88173
-10.39	0.18	0.00	0.81	0.00	0.30739	23.85602	43.03047
-10.65	0.17	0.00	0.77	0.00	0.325189	22.14486	40.50398
-10.90	0.17	0.00	0.75	0.00	0.342834	20.63172	38.25287
-11.15	0.16	0.00	0.72	0.00	0.360315	19.28654	36.23691
-11.38	0.16	0.00	0.69	0.00	0.377622	18.0848	34.42306
-11.62	0.15	0.00	0.67	0.00	0.394748	17.0063	32.78399
-11.84	0.15	0.00	0.65	0.00	0.411687	16.03431	31.29692
-12.06	0.14	0.00	0.63	0.00	0.428438	15.15488	29.9427

-12.27	0.14	0.00	0.61	0.00	0.444995	14.35627	28.70519
-12.48	0.13	0.00	0.59	0.00	0.46136	13.62858	27.57067
-12.68	0.13	0.00	0.58	0.00	0.47753	12.96338	26.52741
-12.88	0.13	0.00	0.56	0.00	0.493506	12.35348	25.56535
-13.07	0.12	0.00	0.54	0.00	0.509288	11.7927	24.6758
-13.26	0.12	0.00	0.53	0.00	0.524879	11.27572	23.85123
-13.45	0.12	0.00	0.52	0.00	0.540279	10.79792	23.0851
-13.63	0.11	0.00	0.50	0.00	0.555491	10.35529	22.37167
-15.23	0.09	0.00	0.40	0.00	0.697812	7.264558	17.27171



f 141 = R1
npact

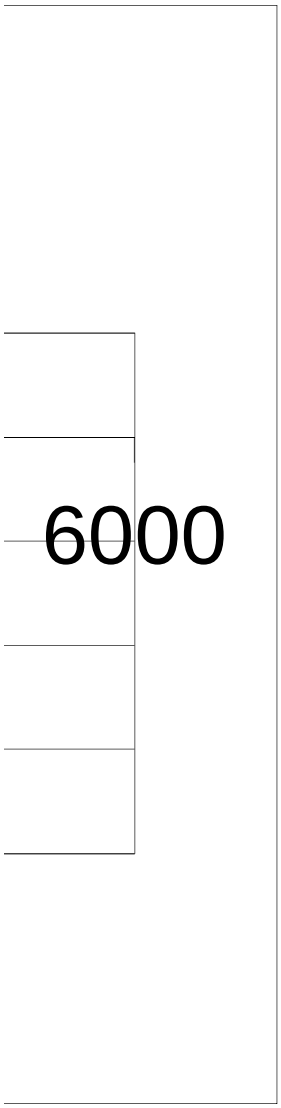
Does R1 combine increase in wwtp and landscape minue c
If so R1 adjsuted to show irrigation returns from new use of

R3
Grove Irrigation Recharge (dec)

dist5	7400
Q5	0
	0

WL change	u
0.00	0.449
0.00	16.395
0.00	10.930
0.00	8.198
0.00	4.099
0.00	2.733
0.00	2.049
0.00	1.640
0.00	1.366
0.00	1.171
0.00	1.025
0.00	0.911
0.00	0.820
0.00	0.745
0.00	0.683
0.00	0.631
0.00	0.586
0.00	0.547
0.00	0.512
0.00	0.482
0.00	0.449
0.00	0.431
0.00	0.410
0.00	0.390
0.00	0.373
0.00	0.356
0.00	0.342
0.00	0.328
0.00	0.315
0.00	0.304
0.00	0.293
0.00	0.283
0.00	0.273
0.00	0.264
0.00	0.256

0.00	0.248
0.00	0.241
0.00	0.234
0.00	0.228
0.00	0.222
0.00	0.216
0.00	0.210
0.00	0.205
0.00	0.164



lecrease in groves? = 52-20 = 32
212 diversions = 139 more return flows

Std Expansion (u<.05)

	W(u)	u
	-2.48E+12	100
	-127.26	10
	0.219	1
	0.560	0.5
10!	1.823	0.1
expansion	4.038	0.01
	6.332	0.001
	8.633	0.0001

	-2.88	10
	-0.58	1
	0.116	0.5
2 term	1.725	0.1
approx	4.028	0.01
(jacob)	6.331	0.001
	8.633	0.0001

Large u (1 < u < infinity)

u	W(u)	numer.	denmm.
100	3.68E-46	1.09E+08	1.1E+08
10	4.16E-06	20465.84	22351.54
1	0.219	36.53486	61.26411
0.5	0.560	10.23407	22.17564
0.1	1.886	1.320509	6.334455
0.01	8.446	0.355932	4.172059
0.001	69.390	0.276423	3.979615
0.0001	678.197	0.268634	3.9606

<<very close at .5
<<below range, but close
..then blows up

<< 2 term not so good at .1, but close

Number-Theoretic Significance of li (x)

(Assuming Riemann's hypothesis that all non-real zeros of $\zeta(z)$ have a real part of $\frac{1}{2}$)

5.1.50 $\text{li}\,(x)-\pi(x)=O(\sqrt{x}\ln x) \qquad (x\to\infty)$

$\pi(x)$ is the number of primes less than or equal to x.

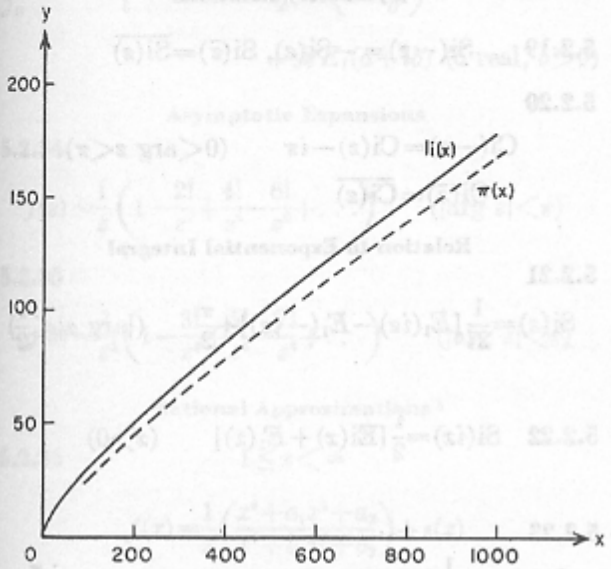


FIGURE 5.5. $y=\text{li}(x)$ *and* $y=\pi(x)$

Asymptotic Expansion

5.1.51

$$E_n(z)\sim\frac{e^{-z}}{z}\left\{1-\frac{n}{z}+\frac{n(n+1)}{z^2}-\frac{n(n+1)(n+2)}{z^3}+\ldots\right\}$$

$(|\arg z|<\frac{3}{2}\pi)$

Representation of $E_n(x)$ for Large n

5.1.52

$$E_n(x)=\frac{e^{-x}}{x+n}\left\{1+\frac{n}{(x+n)^2}+\frac{n(n-2x)}{(x+n)^4}+\frac{n(6x^2-8nx+n^2)}{(x+n)^6}+R(n,x)\right\}$$

$$-.36n^{-4}\le R(n,x)\le\left(1+\frac{1}{x+n-1}\right)n^{-4} \quad (x>0)$$

Polynomial and Rational Approximations [5]

5.1.53 $0\le x\le 1$

$$E_1(x)+\ln x=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5+\epsilon(x)$$

$$|\epsilon(x)|<2\times10^{-7}$$

$a_0=-.57721\ 566$ $\quad a_3=\ .05519\ 968$

$a_1=\ .99999\ 193$ $\quad a_4=-.00976\ 004$

$a_2=-.24991\ 055$ $\quad a_5=\ .00107\ 857$

5.1.54 $0\le x<\infty$

$$e^xE_1(x)=\frac{x^4+a_1x^3+a_2x^2+a_3x+a_4}{x^4+b_1x^3+b_2x^2+b_3x+b_4}+\epsilon(x)$$

$$|\epsilon(x)|<5\times10^{-5}$$

$a_1=\ 8.57332\ 87418$ $\quad b_1=\ 9.57332\ 23354$

$a_2=18.05901\ 69730$ $\quad b_2=25.63295\ 61478$

$a_3=\ 8.63476\ 08905$ $\quad b_3=21.09965\ 30827$

$a_4=\ .26777\ 37350$ $\quad b_4=\ 3.95849\ 69205$

5.1.55 $0\le x<\infty$

$$xe^xE_1(x)=\frac{x^4+a_1x^3+a_2x^2+a_3x+a_4}{x^4+b_1x^3+b_2x^2+b_3x+b_4}+\epsilon(x)$$

$$|\epsilon(x)|<2\times10^{-8}$$

$a_1=\ 8.57332\ 87401$ $\quad b_1=\ 9.57332\ 23454$

$a_2=18.05901\ 69730$ $\quad b_2=25.63295\ 61486$

$a_3=\ 8.63476\ 08925$ $\quad b_3=21.09965\ 30827$

$a_4=\ .26777\ 37343$ $\quad b_4=\ 3.95849\ 69228$

5.1.56 $1\le x<\infty$

$$xe^xE_1(x)=\frac{x^4+a_1x^3+a_2x^2+a_3x+a_4}{x^4+b_1x^3+b_2x^2+b_3x+b_4}+\epsilon(x)$$

$$|\epsilon(x)|<2\times10^{-8}$$

$a_1=\ 8.57332\ 87401$ $\quad b_1=\ 9.57332\ 23354$

$a_2=18.05901\ 69730$ $\quad b_2=25.63295\ 61486$

$a_3=\ 8.63476\ 08925$ $\quad b_3=21.09965\ 30827$

$a_4=\ .26777\ 37343$ $\quad b_4=\ 3.95849\ 69228$

[5] The approximation 5.1.53 is from E. E. Allen, Note 169, MTAC 8, 240 (1954); approximations 5.1.54 and 5.1.56 are from C. Hastings, Jr., Approximations for digital computers, Princeton Univ. Press, Princeton, N.J., 1955; approximation 5.1.55 is from C. Hastings, Jr., Note 143, MTAC 7, 68 (1953) (with permission).

$\text{Cin}(z)$. We have

$$\text{Cin}(z)=-\text{Ci}(z)+\ln z+\gamma.$$

[7] The notations $\text{Sih}(z)=\int_0^z \sinh t\ dt/t$, $\text{Cinh}(z)=\int_0^z(\cosh t-1)dt/t$ have also been proposed [5.14.]